KHATRA ADIBASI MAHAVIDYALAYA

INTERNAL ASSESSMENT 6TH SEM 2020

SUB: MATH (H) COURSE TITLE : METRIC SPACE AND COMPLEX ANALYSIS

PAPER - SH/MTH /601/C-13 F.M.-10

Answer any one question (10 marks)

1. Let Λ be the domain $\{w|\text{Im}w < \pi\}$. Denote the two components of the boundaryof Λ by $\Gamma_1 = \{w|\text{Im}w = 0\}$ and $\Gamma_2 = \{w|\text{Im}w = \pi\}$. Let C be an arbitrary realconstant.

(a) Verify that the function

$$T = \text{Im}\left(\frac{w}{\pi} + C\cosh w\right)$$

is harmonic on Λ, and satisfies the Dirichlet boundary conditions

$$T\Big|_{\Gamma_1} = 0, \quad T\Big|_{\Gamma_2} = 1.$$

(b) For what values (if any) of C is T a bounded function on Λ, i.e. for what values of C does there exist an $M > 0$ such that $|T(w)| \leq M$ for all $w \in \Lambda$?

2. Evaluate the following integrals, justifying your procedures. For (c) and (d) you should also state why the integral is well defined (i.e., independent of the path taken).

(a) $\int_C \frac{2dz}{z^2-1}$, where C is the circle with radius 1/2, centre 1, positively oriented;

(b) $\int_C \left(e^z + \frac{1}{z}\right) dz$, where C is the lower half of the circle with radius 1, centre 0, negatively oriented;

(c) $\int_C ze^{z^2} dz$;

(d) $\int_C \cosh z dz$.

3. Suppose we have an otherwise entire function with poles at 1 and 2i. Given a power series for this function about the origin, where does it converge? How many power series are there? Why must there be more than one? How can we compute the coefficients for each? What happens if we also allow negative powers; where does the Laurent series converge in that case?

....